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Relationship between the mass of a Schwarzschild Black Hole and the frequency of Hawking Radiation emitted

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ABSTRACT

This research paper aims to understand how an increase in the mass of a Schwarzschild Black Hole affects the amount of Hawking Radiation emitted. In order to create a hypothesis, a theoretical and mathematical relationship between the two variables was derived. Several physics concepts including Schwarzschild radius, Blackbody Radiation and the Uncertainty principle along with certain reasonable assumptions were used to create this model. The paper used data collected from AGN Database of Supermassive Black Holes, which was then extrapolated to form graphs to conclude that there is an inversely proportional relationship between the two variables in question with an increase in the mass of a Schwarzschild Black Hole the frequency of Hawking radiation decreases. The paper finds that Hawking Radiation emitted by AGN Supermassive Black Holes correspond to 'long radio waves' in the EM spectrum. The paper further examines the assumptions made while constructing the models used for investigating the hypothesis and addresses their impact on the conclusions drawn and data analysis conducted.

Keywords**—** *Schwarzschild Black Hole, Hawking Radiation, AGN Supermassive Black Hole Database, Mass, Frequency of radiation*

1. INTRODUCTION

The existence of black holes has been contended for a prolonged period in physics history. Classical notions of the object suggested its escape velocity as greater than speed of light – resulting in the object having significant gravitational field strength. Modern frameworks in physics - quantum mechanics and relativity offer more detailed descriptions of black holes' properties.

The research question of this essay is "What is the relationship between mass of Schwarzschild black hole and frequency of Hawking Radiation?" The paper has a brief introduction of black holes followed by relevant physics concepts. The investigation used data collection using the AGN (Active Galactic Nuclei) database for data on mass of Supermassive Black Holes. The data has been extrapolated to form graphs in order to visualize and quantify the mathematical relationship that exists between the two variables – frequency f and Mass M_{BH}.

Hawking's conjecture was a landmark in the development of Quantum Field Theory (combination of General Relativity and Quantum Mechanics) and the understanding of Black Holes. It assisted in creating laws of black hole dynamics as analogous to the laws of classical thermodynamics, which provided further insight into properties of black hole than the "no hair theorem". Furthermore, this topic merges astrophysics, quantum mechanics (blackbody radiation, uncertainty principle) and relativity. Hence, it is indeed worthy of investigation.

2. BACKGROUND THEORY

2.1 Describing Black Holes

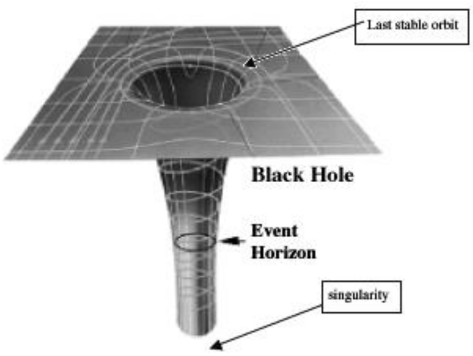

Fig. 1: Black Hole Schematic (made using google draw)

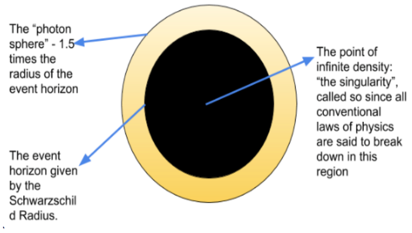

Fig. 2: Black Hole curving spacetime (Slideplayer)

The general black hole is described in terms of the singularity, event horizon and photon sphere. The singularity, as shown in figure 2 is infinitely dense and causes conventional laws of physics to break down. The event horizon is the region of significant gravitational field strength of the black hole – no matter from inside the event horizon can ever escape. The photon sphere is a region of space where there is continuous orbital motion by photon due to the black hole's gravitational force – this motion lasts till they gradually spiral into the blackhole.

2.2 Classifying Black Holes

There are four different types of Black Holes. Each type has different properties and the different equations for their description. Different combinations of 2 properties (electric charge and angular momentum) describe each Black Hole (Table 2.)

Table 1: Table showing classification of Black Holes by Mass

Type of Black Hole	Mass (in terms of M_{sol})
Primordial Black Holes	$M < 1$
Stellar Mass Black Holes	$4 < M < 15$
Intermediate Mass Black Holes	$10^3 < M < 10^4$
Supermassive Black Holes	$10^6 < M < 10^9$

Table 2: Table showing classification of black hole by type [20]

Type of black hole	Mass (in kgs)	Angular momentum (in kgms^{-1})	Charge (in coulombs)
Kerr black hole	$M > 0$	$J > 0$	$Q = 0$
Kerr-Newman black hole	$M > 0$	$J > 0$	$Q \neq 0$
Reissner-Nordström blackhole	$M > 0$	$J = 0$	$Q \neq 0$
Schwarzschild black hole	$M > 0$	$J = 0$	$Q = 0$

2.3 Newtonian representation of a black hole

The idea of a Newtonian black hole comes from the notion of escape velocity with respect to earth's gravitational field. Sufficient work must be done on an object such that when it is launched radially outward from earth's gravitational field, the gravitational force on the object is overcome by its kinetic energy [10] Thus in theory, the velocity of the object will allow it to escape earth's gravitational field into infinity.

Modeling this mathematically, the gravitation force's magnitude is given by the following equation;

$$F_g = \frac{GMm}{r^2} \quad \text{Eq (1)}$$

Where F_g is gravitational force in N (newton)
G is gravitational constant (6.67 x 10^{-11}) in $Nm^{-2}kg^{-2}$
- m, M, r are the masses of test mass, and mass and radius of planet respectively in SI units

The gravitational force between two masses is expressed as directly proportional to their product and inversely proportional to their radial distance squared. In order for test mass $'m'$ to escape the gravitational field of planet with mass $'M'$, initial kinetic energy supplied must equal the final gravitational potential energy (energy needed to escape a gravitational field into infinity). The initial GPE on the surface of the mass is $-\frac{GMm}{R}$ and the final GPE at infinity is 0.

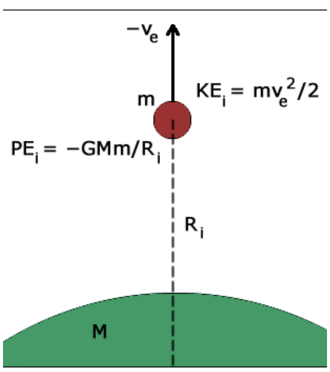

Fig. 3: Energy conservation during test mass launch [3]

Assuming principle of energy conservation holds true,

$$initial\ KE - final\ KE = final\ GPE - initial\ GPE$$

$$\frac{1}{2}mv^2 = 0 - \left(-\frac{GMm}{R}\right)$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

Giving.

$$\frac{1}{2}v^2 = \frac{GM}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} \qquad \text{Eq (2)}$$

where v is escape velocity for test mass in ms^{-1}.

The classical definition of a black hole suggests that the escape velocity for an object must be larger than the speed of light, which is not possible according to the theory of special relativity. However, using Eq(2), the radius of Newtonian Black Hole can be estimated [10]. Since the escape velocity v is greater than speed of light, c.

$$v > c$$

Since,

$$c < \sqrt{\frac{2GM}{R}}$$

Thus,

$$R < \frac{2GM}{c^2}$$

The above equation is used to determine the Schwarzschild radius- minimum radius an object requires in order for light to be unable to escape its gravitational field. A physicist, Karl Schwarzschild, derived the same using theory of general relativity.

$$\therefore R_s = \frac{2GM}{c^2} \qquad \text{Eq (3)}$$

where R_s is Schwarzschild Radius (in m) for a given planet M, in order to have an escape velocity equal to the speed of light $(v_{esc} = c)$

2.4 Blackbody Radiation

A Blackbody is defined as a perfect absorber of all radiation and in theory, an equally efficient emitter of radiations across the electromagnetic spectrum. In order to understand the process, it is useful to apply the quantum theory, which states that energy travels in discrete packets. Assume a material, composed of electrons, which are bound to particles of that particular substance – each electron having an associated wave function depending on which quantum shell it exists in. When electromagnetic radiation comes into contact with the material, electrons are said to oscillate in response to an external electromagnetic field (each electron has a corresponding wave function that can describe its energy). This oscillation only occurs at certain frequencies, meaning that at one time only fixed quantities of energies are absorbed by the electrons [7]. Consequently, the electrons now oscillating at higher frequencies possess on average a higher kinetic energy, resulting in an increase in temperature of the substance. In order for the electrons to radiate on the other hand, the electrons continuously collide with particles of the substance, which results in loss of kinetic energy; hence frequencies of EM waves are emitted [7].

Furthermore, by the first law of thermodynamics it can be concluded that a body is as good an emitter of radiation as it is an absorber. Assuming a body absorbs radiation more efficiently than it emits and is kept in a physical system at the same initial temperature, it would continuously absorb energy from the surroundings without radiating the same amount back to the room [7]. This would never enable the body to exist at thermal equilibrium with its surroundings. Thus, a body must emit as well as it absorbs radiation.

Black holes' property of being perfect absorbers of radiation means they can be modeled as black bodies. However, in their case, no matter or radiation is emitted at all - when something passes the event horizon, it is no longer able to escape the black holes' gravitational field.

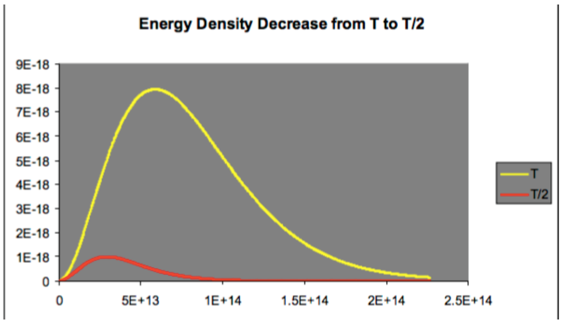

Fig. 4: Illustrating Wien's displacement law [7]

Wien's displacement law states that emitted frequency having maximum energy density is proportional to the temperature of the radiating body:

$$f_{max} \propto T$$

Energy density refers to energy contained by photons per unit volume of the radiator [7][19]. A black body radiates all wavelengths across the electromagnetic spectrum, however the most "abundant" or dense with regards to energy (and thus, number of photons) is proportional to the temperature. Consider figure 4, the frequencies to the left of f_{max} correspond to release of low-energy photons from low energy discrete levels. Although large numbers of photons will be released, since the energy per photon is less, the total energy would not be maximised. At f_{max}, total energy carried by the photons is maximised (the frequency is high hence the energy per photon is greater and also there are sufficient photons released in order to allow maximum total energy). At greater temperatures, the value of f_{max} is higher to due molecules possessing higher K.E on average thereby allowing the electrons to emit photons of higher energy.

2.5 Heisenberg Uncertainty Principle

The uncertainty principle states there will always be an uncertainty associated with the measurement of two conjugate quantities – displacement and momentum. This is inherent due to the particle –wave nature of matter [8]. Particles show wave like properties as they undergo diffraction (as shown in the electron diffraction experiment). Consider the electrons' motion as shown in Figure 5 (a): the slit width is broad and hence the uncertainty associated with the position of electron along the distance of the slit (Δx) is large. However, momentum of the particles is directed in the x-axis (horizontal plane), thus there is negligible uncertainty in momentum. In Figure 5 (b) 2, the slit is considerably narrowed meaning the position of electrons can be more accurately determined (Δx is less) while the electrons velocity is directed in slightly different directions [8]. Hence, there is a component of their momentum acting in the y-axis (seen in fig.5 part (b))

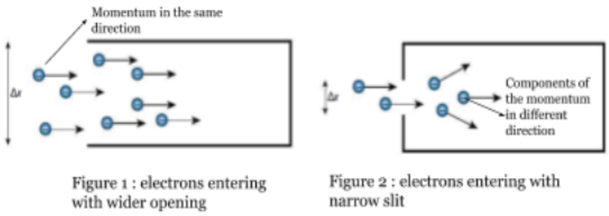

Fig. 5: Heisenberg Uncertainty Principle electron diffraction experiment (Hamper)

The probable position of a particle thus is given by a wave packet (Figure 6)- superimposed waves that have areas of constructive interference showing the likely positions of the particle at a particular given time. Combining several waves of different wavelengths together will result in an interference pattern showing an area of constructive interference, localizing the position of the particle. This is because having a distinct wavelength of the particle would mean having certainty of its momentum, causing significant uncertainty in finding its position [14].

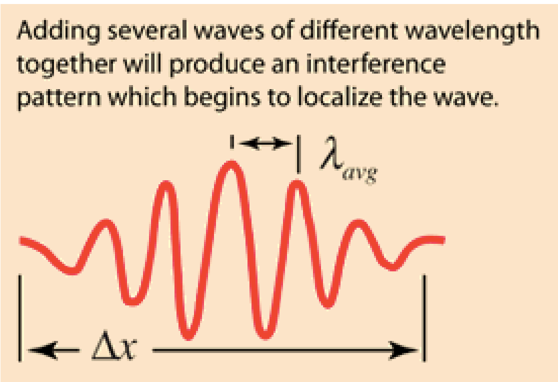

Fig. 6: Localising the probability wave function [13]

The relationship between displacement and momentum can also be expressed in terms of uncertainty in the energy of particle and the time period for which it contains that energy.

$$\boxed{\Delta x \Delta p \geq \frac{h}{4\pi}} \quad \text{Eq (4)}$$

Since kinetic energy of a particle can be expressed in terms of momentum. The assumption here is the particle has no potential energy (not in any field, no other sources of potential energy)

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{p^2}{m}\right)$$

Differentiating both sides with respect particle's momentum,

$$\frac{d}{dp}E = \frac{d}{dp}\left(\frac{p^2}{m}\right)$$

Since this allows to find a relationship between change in energy

$$\frac{\Delta E}{\Delta p} = \frac{1}{2} \times \frac{2p}{m}$$

Rearranging,

$$dE = \frac{\Delta p \times p}{m} = \frac{m \times v \times \Delta p}{m} = v \times \Delta p = \frac{\Delta x \times \Delta p}{\Delta t}$$

$$\Delta E = \frac{\Delta x \times \Delta p}{\Delta t}$$

$$\Delta E \times \Delta t = \Delta x \times \Delta p$$

Substituting in

$$\Delta E \Delta t \geq \frac{h}{4\pi} \quad \text{Eq (5)}$$

Hence, the above expression suggests that for a particle with short lifespan Δt the energy contained within them may vary considerably [13]. This allows for production of virtual particles pairs (particles that come into existence for short time periods in a field) to occur in a vacuum, provided their short lifespan is Δt. The production occurs in particle-antiparticle pairs, one of which must carry positive energy while the other - negative energy to allow for annihilation.

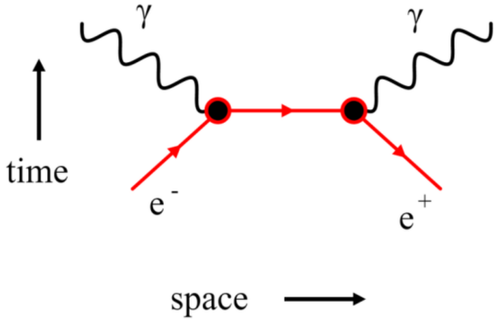

Fig. 7: Example of particle antiparticle (electrons positron pair) annihilating each other to emit two gamma ray photons [15]

2.6 The mechanism of Hawking radiation

Classical black holes are considered perfect absorbers, which would mean they would completely absorb any radiation across the electromagnetic spectrum and emit none whatsoever. An object, which emits no radiation, therefore, was to have no temperature. However, the uncertainty principle allows for virtual pair production in vacuum conditions as long as their lifetime was for a time:

$$\Delta t \geq \frac{h}{4\pi(2\Delta E)}$$

Where $2\Delta E$ is the energy contained within two photons;
Substituting for energy of photon by $E = hf$, gives

$$\Delta t = \frac{1}{8\pi f} \quad \text{Eq (6)}$$

Where Δt is time taken for creation and annihilation of the pair, and f is the frequency of photons.

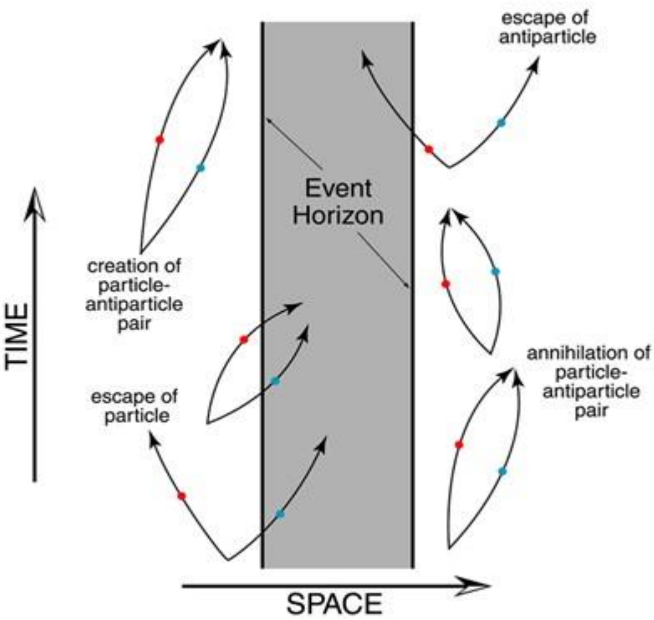

Fig. 8*: Virtual Pair production beyond event horizon and Hawking Radiation [2]*

Hawking stated that in the small instant that these particles are created and destroyed, as given in Eq(6) - if one virtual particle out of the pair, due to gravitational field strength of the black hole is absorbed by it, the corresponding virtual particle in the pair (since no longer having its antiparticle for the annihilation process) is released from the black hole (*McGill*). Since the initial total energy in the vacuum was zero, the total energy of one of the particles (rest mass + kinetic energy) will be negative. Particles with negative energy cannot exist outside the event horizon except for time period Δt - these must undergo annihilation together with their positive energy counterpart [18]. Hence, the positive energy particle (particle or antiparticle) may not enter the event horizon. The negative particle however can exist inside the event horizon, allowing its positive counterpart to escape to infinity.

Absorbing the particle with negative energy results in the black hole losing energy equivalent in magnitude to energy carried to infinity by the positive energy particle (refer Figure 8) and corresponding mass ($\because E = mc^2$). The opposite of this process cannot occur since negative energy particles cannot exist independently (without there positive counterpart) [18].

2.6 Derivation of equation for Temperature

The energy of a photon E is proportional to its frequency, *f, where h,* Planck's constant is the constant of proportionality.

$$\boxed{E = hf} \qquad \text{Eq (7)}$$

Since speed of light is, $c = f\lambda$

where *f* is frequency of photons and λ, wavelength of light.

$$\boxed{E = \frac{hc}{\lambda}} \qquad \text{Eq (8)}$$

From the mass-energy equivalence principle $E = mc^2$, we conclude that a change in mass will result in a corresponding change in energy. Therefore, $\Delta E = \Delta mc^2$

Rearranging,

$$\frac{\Delta E}{c^2} = \Delta m$$

To represent the energy contained in one photon, we can substitute Eq (8) to give

$$\Delta m = \frac{hc}{\lambda c^2} = \frac{h}{\lambda c}$$

The wavelength of the Black Hole is assumed to be of the order of Schwarzschild radius, $\lambda \approx R_s$

According to Wien's Displacement Law for Blackbody (refer Blackbody radiation):

$$\because f_{max} \propto T \rightarrow \lambda_{max} \propto \frac{1}{T}$$

Specifically,

$$\lambda_{max} T = 2.9 \times 10^{-3} mK$$

The lower the temperature of blackbody (T),the higher the wavelength radiated (λ_{max}) [8]. Since blackholes have exceedingly low temperatures, emitted wavelength is significantly large – hence, it can be assumed that the order of magnitude of $R_s \approx \lambda$.

Thus,

$$\Delta m = \frac{h}{R_s c}$$

Since, $R_s = \frac{2GM}{c^2}$ giving,

$$\boxed{\Delta R_s = \frac{2G\Delta M_{BH}}{c^2}} \qquad \text{Eq (9)}$$

The change in mass of black hole - ΔM has taken place by the black hole absorbing a photon with energy, $E = hf$. Gaining energy from absorption of a photon, the total energy the black hole now contains includes the energy of the photon.

Eq(9) suggests how R_s will change with a change in M_{BH}. Since the photon transfers its energy to the Black Hole, we can substitute into Eq(9).

$$\Delta R = \frac{2(\frac{h}{\lambda c})G}{c^2} = \frac{2Gh}{\lambda c^3} = \frac{2Gh}{R_s c^3}$$

Rearranging, $R_s \Delta R = \frac{2Gh}{c^3}$ [9]

The surface area of the black hole is given by,

$$A = 4\pi {R_s}^2$$

Differentiating both sides,

$$\frac{dA}{dR} = 8\pi R_s$$

$$dA = 8\pi R_s dR$$

$$dA = 8\pi \frac{2Gh}{c^3}$$

This suggests that the area increases as a consequence of putting one photon in the black hole. Since photons are being considered as the unit for entropy. (addition of one photon gives a ΔS of 1).

Therefore, $dA = 8\pi \frac{2Gh}{c^3} dS$. Integrating both sides yields,

$$\int dA = \int 8\pi \frac{2Gh}{c^3} dS$$

$$A = 16\pi \frac{Gh}{c^3} S$$

Hence, the formula can be rearranged to give entropy

$$S = \frac{Ac^3}{16\pi Gh}$$

Since,

$$dE = TdS$$

In the case of the addition of one photon, dS would be 1.

$$dE = T = \frac{hc}{\lambda}$$

$$= \frac{hc}{R_s}$$

Thus,

$$\boxed{T = \frac{hc^3}{2MG}} \qquad \text{Eq (10)}$$

Giving the relationship $T\alpha\frac{1}{M}$ [9]

The derivation above is a much more simplistic version than that Hawking provided in order to derive Temperature of the Black Hole (T_{BH}) as follows,

$$\boxed{T = \frac{hc^3}{16\pi k_B GM}} \qquad \text{Eq (11)}$$

where $\bar{h}$ is a form of Planck's constant and c is the speed of light. G *and* k_B represent the universal constant of gravitation (in a relativistic sense, referring to the curvature of spacetime) and the Boltzmann constant respectively. [10] Since,

$$k_B T = \frac{hc^3}{16\pi GM}$$

$$E = \frac{hc^3}{16\pi GM}$$

The energy of a photon can be represented as $E = hf$. Substituting in the above equation gives [10],

$$\boxed{f = \frac{c^3}{16\pi GM}} \qquad \text{Eq (11)}$$

Giving following inverse relationship,

$$f\alpha\frac{1}{M}$$

Thus, a black hole with larger mass is said to emit radiation of lower frequencies. [10].

3. METHODOLOGY OF DATA COLLECTION

The AGN Black Hole Mass Database was used as the source for collecting values of the masses of Supermassive Black Holes in the nearby universe (gravitational redshift, z, $z < 0.1$ for most objects) [5]. The database used a process called reverberation mapping. The processes can be simply described as measuring the time delay from emissions from the acretion disk of the black hole (region where matter circularly orbiting the black hole radiates heat due to immense friction generated by their circular motion) and emissions from the Broad Line Region (BLR) which is larger in diameter than the acretion disk*)* [5] . The calculation used by the source has been described in Appendix 1 along with exhaustive sample of data collected. The mass of the objects was given in terms of $log\left(\frac{M_{BH}}{M_{sol}}\right)$, where $\frac{M_{BH}}{M_{sol}}$ is the mass of the Black Hole expressed in terms of solar mass units. The values were processed to give M_{BH} in kilograms and calculate a corresponding uncertainty for the data obtained.

Table 3: Sample data in the form of data table [6]

Object ID	$\boldsymbol{log\left(\frac{M_{BH}}{M_{sol}}\right)}$	Mass ($\boldsymbol{M_{sol}}$)	$\boldsymbol{M_{BH}(in\ kgs)}$	Uncertainty ($\boldsymbol{in\ kgs}$)
Mrk335	7.230	1.70E+07	3.38E+37	3.33E+36
Mrk1501	8.067	1.17E+08	2.32E+38	7.32E+37
PG0026+129	8.487	3.07E+08	6.10E+38	1.48E+38
PG0052+251	8.462	2.90E+08	5.76E+38	1.16E+38
Fairall9	8.299	1.99E+08	3.96E+38	8.70E+37
Mrk590	7.570	3.71E+07	7.39E+37	1.15E+37
3C120	7.745	5.55E+07	1.11E+38	9.90E+36
H0507+164	6.876	7.51E+06	1.50E+37	7.74E+36
Ark120	8.068	1.17E+08	2.33E+38	2.90E+37
Mrk6	8.102	1.26E+08	2.52E+38	2.25E+37

3.1 Sample Calculations

Mass in M_{sol} :

$$\because\ log\left(\frac{M_{BH}}{M_{sol}}\right) = 7.230$$

$$\frac{M_{BH}}{M_{sol}} = 10^{7.230} = 1.7\ \times 10^7 M_{sol}$$

Mass in $M_{BH}(kgs)$:

$$\because\ mass\ of\ the\ sun\ or\ 1M_{sol} = 1.9 \times 10^{30} kgs$$

$$M_{BH}(in\ kgs) = 1.7\ \times 10^7 M_{sol} \times\ 1.9 \times 10^{30} kgs = 3.38 \times 10^{37}$$

Uncertainty $(in\ kgs)$

Given, $\log\left(\frac{M_{BH}}{M_{sol}}\right) = 7.230 \pm 0.043$

$$\because \Delta logx = \frac{\log(x + \Delta x) - \log(x - \Delta x)}{2}$$

Let $\left(\frac{M_{BH}}{M_{sol}}\right) = x$, which is a known value. Substituting this in the above equation shall allow us to find the uncertainty associated with the ratio of $\frac{M_{BH}}{M_{sol}}$.

$$\Delta\log\left(\frac{M_{BH}}{M_{sol}}\right) = \frac{\log(1.7\times10^{7}+\Delta x)-\log(1.7\times10^{7}-\Delta x)}{2}$$

$$0.043 = \frac{\log(1.7\times10^{7}+\Delta x)-\log(1.7\times10^{7}-\Delta x)}{2}$$

Using numerical solve tool GDC Calculator (Ti-inspire) , $\Delta x = 1.68\times10^{6}\ M_{sol}$

This was multiplied by the $M_{sol} = 1.9\times10^{30}\ kgs$ to give uncertainty of M_{BH} in kilograms.

$$\Delta M_{BH} = 1.68\times10^{6}\ M_{sol}\times1.9\times10^{30} kgs$$

4. ANALYSIS

Two graphs plotted to ascertain the mathematical relationship between the two variables. The first one is a graph of f vs M_{BH}, while the second is a graph of $f\ vs\ \frac{1}{M_{BH}}$.

The graph in Figure 9 shows a non-linear correlation between the frequency of the radiation and the mass of a Black hole. The curve of best fit (which is of the form of $\frac{A}{M}, where\ A\ is\ a\ constant$) mathematically shows that as values of the mass increase, the value of frequency decreases proportionally. From the shape of the graph, the degree of the relationship cannot be deduced (if the relationship is inverse, inverse–square etc.). The values of frequency have been calculated using $Eq(11)$ and lie perfectly on the line of best fit. The graph asymptotes towards the x and $y - axis$, suggesting that as the mass of the black hole gets infinitely larger the frequency radiated becomes increasingly negligible. The gradient of the graph increases as the mass of the black hole decreases – as mass decreases, increase in frequency per unit mass is greater (as suggested by the y-asymptote). A decrease in mass in range of $0\ to\ 5E+37$ will cause a greater increase in frequency radiated than a decrease in mass in range - $1E+38\ to\ 2E+38$. This leads us to conclude that change in frequency per unit mass $i.e\ gradient\ or\ \frac{df}{dM_{BH}}$, increases as the mass of the black hole decreases.

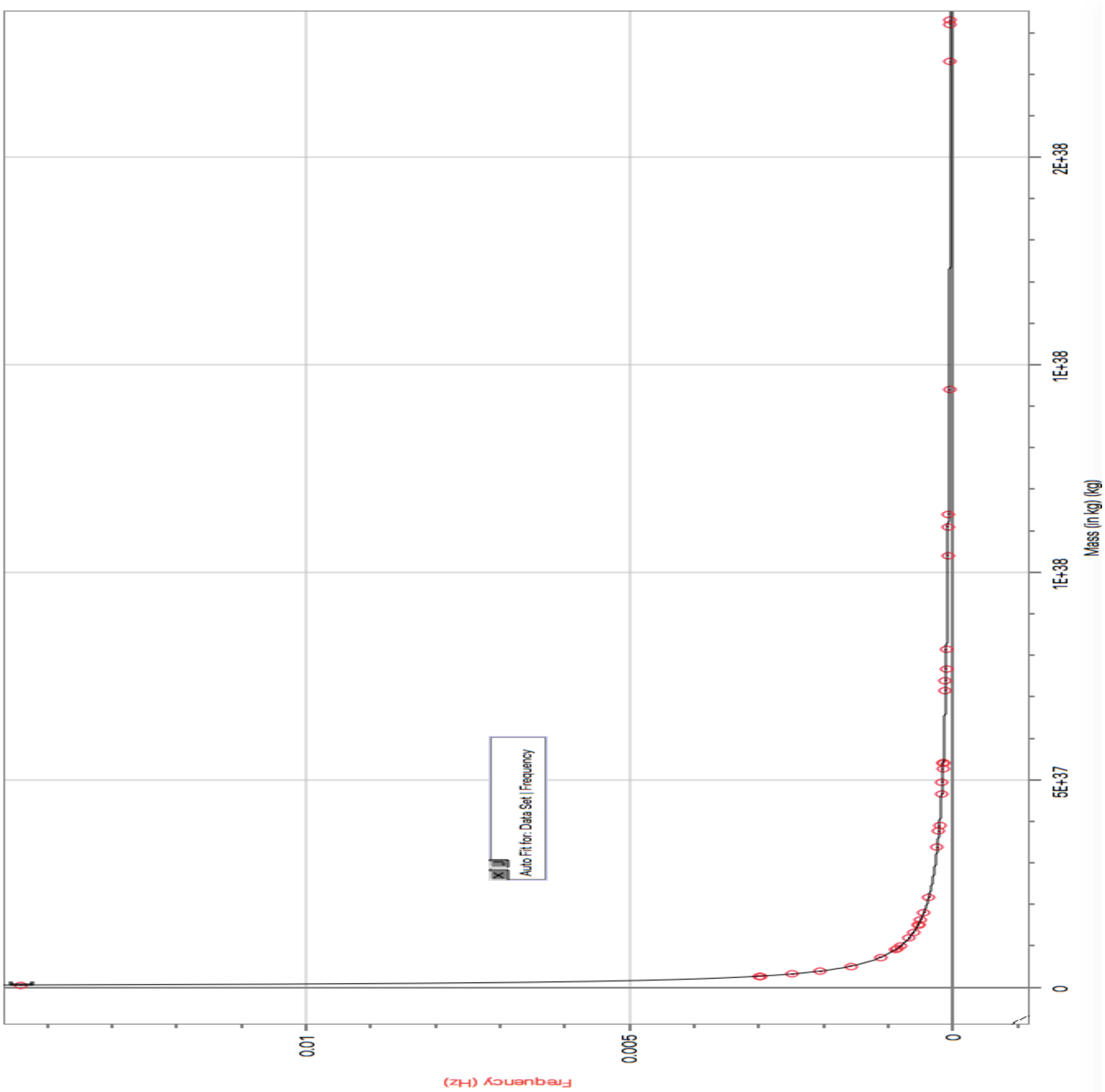

Fig. 9: Graph of frequency of Black Hole (*f*) vs. M_{BH}

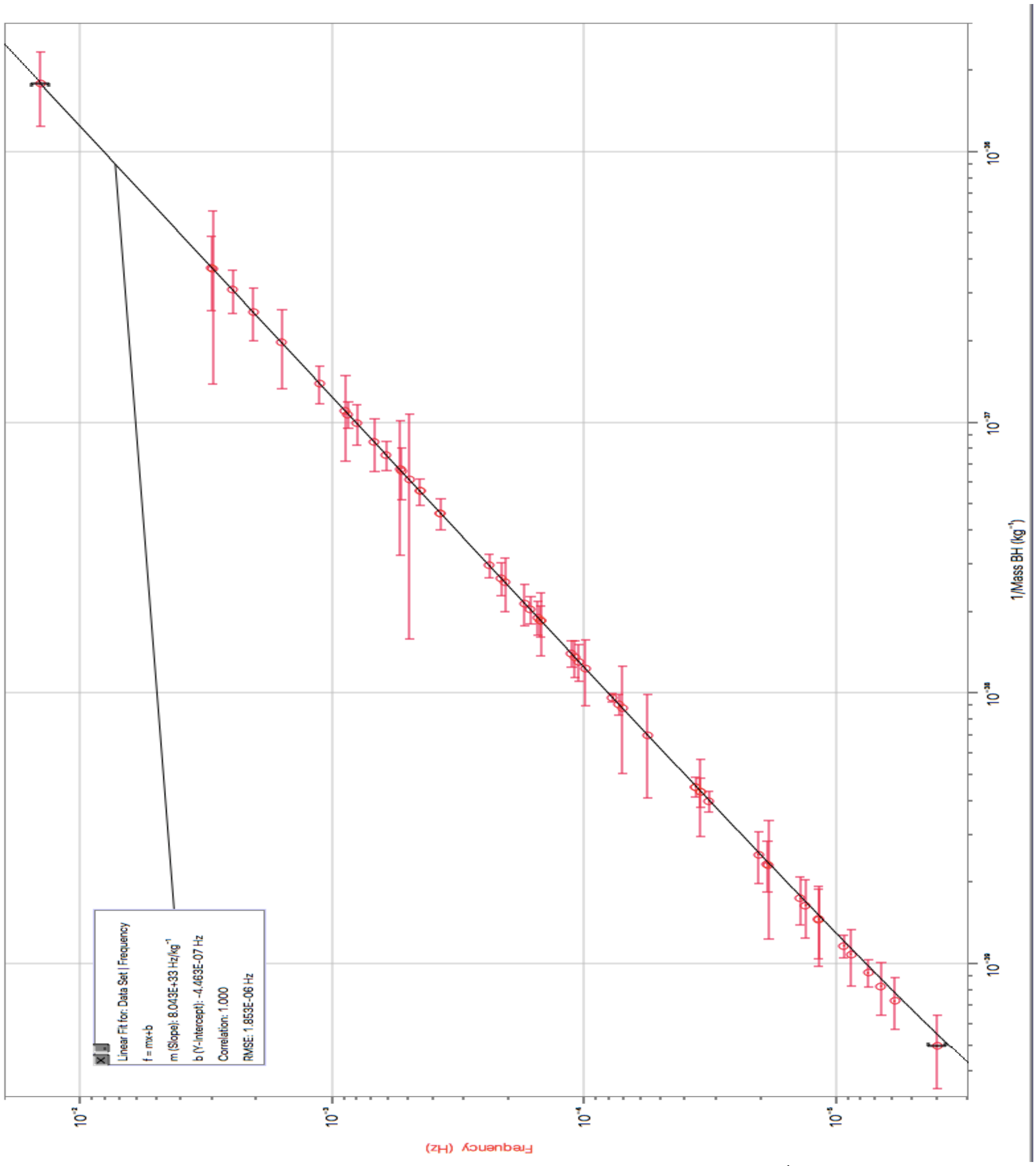

Fig. 10: Graph of frequency of Black Holes (f) vs. 1/Mass ($\frac{1}{M_{BH}}$)

The graph in Figure 10 shows an inversely proportional relationship between frequency ($f$and mass (M_{BH}), justified since the shape of the graph is a straight line. Ideally, the graph should pass through the origin, but the deviation in the y-intercept is likely due to the uncertainties in values of ($\frac{1}{M_{BH}}$). Considering the range shown in graph and data in Appendix 1, the frequencies of the Black Hole likely correspond to *"long radio waves"* ranging from 33 Hz to 33 x 10^3 Hz [10]. Its noteworthy, the frequency calculated represents f_{max}, the peak frequency from the spectrum emitted by blackhole.

5. CONCLUSION

In theory, the relationship between frequency of the radiation and mass of black hole can be justified as follows. The temperature of a black hole is inversely proportional to its mass, as suggested by expression for $T_{BH.}$ An alternative explanation is that, smaller the mass of black hole, shorter the distance that needs to be travelled by the particle with negative energy (in the virtual particle pair) to become a real particle (upon crossing the event horizon). Therefore, the average rate of emission become greater– since it is easier for more negative energy particles to fall into the black hole. A body emitting larger amounts of radiation will thus have higher temperature. Thus, a decrease in mass of black hole will cause an increase in its temperature. Consequentially, resulting in more energetic vacuum fluctuations and hence formation of virtual particles with greater energy (i.e. photons with greater frequency).

The graphs (Fig. 9 and Fig. 10) can be used to conclude an inverse relationship between the mass of Schwarzschild black hole and frequency of emitted radiation exists. The sampling of data was limited to Supermassive black holes (masses of intermediate and stellar ranges were not considered). However, modelling their properties (irrespective of the category of its mass) as Schwarzschild Black Hole, the results can be extended to generalise the inverse relationship between mass and frequency of radiation emitted.

The emitted frequency falls into spectrum of long radio waves and is significantly low explaining why Hawking Radiation is hard to detect. Moreover, interactions with CMB (Cosmic Microwave Background Radiation) spread throughout universe make it more challenging. The inverse relationship, as concluded by the essay, could be a reason for the short existence of Primordial Blackholes – minimum mass estimated as 10^{-8} kg. A black hole of 10^{-8} kg would radiate exceedingly large frequencies, emitting ever increasing values as the black hole continued losing mass due to Hawking Radiation.

6. EVALUATION

The derivation of the Schwarzschild radius was inaccurate because in equation the mass of the test object represent different quantities in a relativistic sense. The mass m associated with the kinetic energy of the object is its rest mass while the mass m in the formula for gravitational potential energy is the relativistic mass – two different quantities according to the theory of relativity [13]. The rest mass is measured by an observer at rest relative to the object in motion while relativistic mass gives a measure of the total energy of object in motion: (total energy = rest mass + mass increase due to K.E) [13]. The equation below shows the different types of mass and can be simplified to exclude the masses of the test mass,

$$\frac{1}{2}m_0v^2 = \frac{GM(\gamma m)}{R}$$

where m_0 is the rest mass of object whereas γm represents the relativistic mass. γ is the Lorenz factor given by expression $\gamma = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$, [13] where v is the speed of the particle and c is the speed of light

Including the relativistic theory, the equation for Newtonian GPE also gets modified

$$E = \left(\frac{-GM}{R} - \frac{1}{4}\frac{R_s{}^2c^2}{r^2} - \ldots\ldots\right)\gamma m$$

Where the gravitational potential is given by the expression in brackets times the relativistic mass (γm) [11]. The difference in the expression gives the correction in gravitational potential for an object (gravity is considered as the curvature of space-time by mass) according to relativity. Detailing that correction is beyond the scope of this essay however, the 1st order correction for GPE calculated using classical Newtonian equation for the earth is about $10^{-4}\%$ however for a black hole the correction is much more significant (about 10%). This did not affect the tabulated results since calculations made use of hawking's formula. However, it makes the derivation for Temperature much less rigorous contributing to inaccuracy.

It must be noted that the process of Hawking Radiation does not take place in isolation, i.e Black Hole evaporation through Hawking Radiation takes place in parallel to the absorption of CMBR (Cosmic Microwave Background Radiation) that exists at a current temperature $2.275°K$ in the universe [16]. This radiation is said to exist as a result of the big bang and exists across all EM spectrum exhibiting characteristic of blackbody radiation. However, it takes approx. 3.17 x 10^{30} for absorption of CMBR to cause a significant change in mass (approx. 10^{38} seconds). Since this number is greater than the age of the universe, the affect of CMBR is negligible. [16]

The conclusions of this paper cannot be generalized to other black holes – Kerr and Kerr-Newman Black Hole for example have angular momentum (J) and angular momentum and charge respectively (J and Q). This is because the temperature for these black holes cannot be modelled by Hawking's formula as used in the paper. Charge and angular momentum both have affect on the black holes temperature hence resulting in a different equation used as a model.

The uncertainty provided by the AGN database was largest for objects with greater gravitational redshift values (for $z > 0.1$). The farther a galaxy from the earth, the greater the gravitational redshift observed in radiation [8] – thus leading to more uncertainty for galaxies located at a farther distance. This uncertainty was carried forward for frequency – black holes at greater distances (to observer/earth) have higher uncertainty for emitted frequencies.

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8. REFERENCES

[1] "Black Body Radiation." Astro.ufl.edu, users-phys.au.dk/fogedby/statphysII/notes/BBR.pdf. "The Wien's Displacement Law"
[2] Black Hole Thermodynamics, nrumiano.free.fr/Estars/bh_thermo.html. "Derivation of Gravitational Escape Velocity Equation." Derivation of Gravitational
[3] Escape Velocity Equation by Ron Kurtus - Physics Lessons: School for Champions, www.school-for-champions.com/science/gravitation_escape_velocity_derivation.htm#.W9cpHHozbq0
[4] "Gravity, Energy, and Light Einstein and Newton Ppt Download." SlidePlayer, slideplayer.com/slide/9630758/. Accessed 7 Sept 2018.
[5] Bentz, Misty C., and Sara Katz. "The AGN Black Hole Mass Databse." The AGN Black Hole Mass Databse, pp. 1–7., arxiv.org/pdf/1411.2596.pdf. "1-3"
[6] Bentz, Misty C., and Sarah Katz. "The AGN Black Hole Mass Database". The AGN Black Hole Mass Database, 2018, http://www.astro.gsu.edu/AG2ass/#acknowledge.
[7] Fowler, Michael. "Black Body Radiation." Black Body Radiation, galileo.phys.virginia.edu/classes/252/black_body_radiation.html. "Pages 1-2,2-3,5-6"
[8] Hamper, Chris. Higher Level Physics. Pearson Education Limited, 2014.

[9] LoPresto, Michael C. "Some Simple Black Hole Thermodynamics." Some Simple Black Hole Thermodynamics, Stanford University, web.stanford.edu/~oas/SI/SRGR/lib/BlackHoleThermoShort.pdf.
[10] Mahto, Dipo et al. Article.Sciencepublishinggroup.Com, 2018, http://article.sciencepublishinggroup.com/pdf/10.11648.j.ijass.20130104.15.pdf.
[11] Marolf, Donald. "Notes on Relativity and Cosmology for PHY312." PHY312, 0 Jan. 2003, web.physics.ucsb.edu/~marolf/MasterNotes.pdf. Sections 6,8,9
[12] Nave, Rob. "Active Galaxies." Active Galaxies, hyperphysics.phy-astr.gsu.edu/hbase/Astro/actgal.html#c2. "Active Galactic Nuclei"
[13] Nave, Rob. "Length Contraction." Time Dilation/Length Contraction, hyperphysics.phy-astr.gsu.edu/hbase/Relativ/tdil.html#c3. "Rest mass/Relativistic Mass"
[14] Nave, Rob. Particle Lifetimes from the Uncertainty Principle. hyperphysics.phy-astr.gsu.edu/hbase/quantum/parlif.html.
[15] Nave, Rod. "The Uncertainty Principle." Uncertainty Principle, hyperphysics.phy-astr.gsu.edu/hbase/uncer.html.
[16] NRAO CV. "Cosmic Microwave Background Radiation." Cosmic Microwave Background Radiation, www.cv.nrao.edu/course/astr534/CMB.html.
[17] Physics.Mcgill.Ca, 2018, http://www.physics.mcgill.ca/~imorrison/GuestLectures/Phys253_BHthermo.pdf.
[18] Steve Carl. "Professor Steve Carlip." Steve Carlip, carlip.physics.ucdavis.edu/#Hawkrad. Hawking Radiation
[19] Victoria, University of. "Chapter 2: Blackbody Radiation." pp. 1–4., astrowww.phys.uvic.ca/~tatum/stellatm/atm2.pdf.
[20] Weisstein, Eric. "Black Hole -- from Eric Weisstein's World of Physics." Scienceworld.wolfram.com, scienceworld.wolfram.com/physics/BlackHole.html.

APPENDIX

Name of Supermassive Black Hole	log M_{BH} (M_{SUN})	M_{BH} (M_{SUN})	M_{BH} (in kgs)	Uncertainty in Mass (kgs)	Frequency of H.R. (Hz)	Wavelength of H.R. (m)
Mrk335	7.230	1.70E+07	3.38E+37	3.33E+36	2.38E-04	1.26E+12
Mrk1501	8.067	1.17E+08	2.32E+38	7.32E+37	3.46E-05	8.66E+12
PG0026+129	8.487	3.07E+08	6.10E+38	1.48E+38	1.32E-05	2.28E+13
PG0052+251	8.462	2.90E+08	5.76E+38	1.16E+38	1.39E-05	2.15E+13
Fairall9	8.299	1.99E+08	3.96E+38	8.70E+37	2.03E-05	1.48E+13
Mrk590	7.570	3.71E+07	7.39E+37	1.15E+37	1.09E-04	2.76E+12
3C120	7.745	5.55E+07	1.11E+38	9.90E+36	7.27E-05	4.13E+12
H0507+164	6.876	7.51E+06	1.50E+37	7.74E+36	5.37E-04	5.58E+11
Ark120	8.068	1.17E+08	2.33E+38	2.90E+37	3.45E-05	8.69E+12
Mrk6	8.102	1.26E+08	2.52E+38	2.25E+37	3.19E-05	9.39E+12
Mrk79	7.612	4.09E+07	8.14E+37	2.22E+37	9.87E-05	3.04E+12
PG0804+761	8.735	5.43E+08	1.08E+39	1.25E+38	7.44E-06	4.03E+13
PG0844+349	7.858	7.22E+07	1.44E+38	5.96E+37	5.60E-05	5.36E+12
Mrk110	7.292	1.96E+07	3.90E+37	8.74E+36	2.06E-04	1.46E+12
PG0953+414	8.333	2.15E+08	4.29E+38	9.23E+37	1.87E-05	1.60E+13
NGC3227	6.775	5.96E+06	1.19E+37	2.63E+36	6.78E-04	4.42E+11
Mrk142	6.294	1.97E+06	3.91E+36	8.59E+35	2.05E-03	1.46E+11
NGC3516	7.395	2.48E+07	4.94E+37	5.54E+36	1.63E-04	1.84E+12
SBS1116+583A	6.558	3.62E+06	7.20E+36	1.17E+36	1.12E-03	2.69E+11
Arp151	6.670	4.68E+06	9.31E+36	1.06E+36	8.63E-04	3.48E+11
NGC3783	7.371	2.35E+07	4.67E+37	8.14E+36	1.72E-04	1.74E+12
Mrk1310	6.212	1.63E+06	3.24E+36	5.90E+35	2.48E-03	1.21E+11
NGC4051	6.130	1.35E+06	2.68E+36	8.24E+35	3.00E-03	1.00E+11
NGC4151	7.555	3.59E+07	7.15E+37	8.03E+36	1.12E-04	2.67E+12
Mrk202	6.133	1.36E+06	2.70E+36	1.69E+36	2.97E-03	1.01E+11
NGC4253	6.822	6.64E+06	1.32E+37	1.62E+36	6.08E-04	4.93E+11
Mrk50	7.422	2.64E+07	5.26E+37	7.51E+36	1.53E-04	1.96E+12
NGC4395	5.449	2.81E+05	5.60E+35	1.71E+35	1.44E-02	2.09E+10
PG1226+023	8.839	6.90E+08	1.37E+39	2.96E+38	5.85E-06	5.13E+13
PG1229+204	7.758	5.73E+07	1.14E+38	4.85E+37	7.04E-05	4.26E+12
NGC4593	6.882	7.63E+06	1.52E+37	3.23E+36	5.30E-04	5.66E+11
NGC4748	6.407	2.55E+06	5.08E+36	1.65E+36	1.58E-03	1.90E+11
PG1307+085	8.537	3.44E+08	6.85E+38	1.95E+38	1.17E-05	2.56E+13
NGC5273	6.660	4.57E+06	9.09E+36	3.16E+36	8.84E-04	3.39E+11
Mrk279	7.435	2.72E+07	5.42E+37	1.41E+37	1.48E-04	2.02E+12
PG1411+442	8.539	3.46E+08	6.89E+38	2.25E+38	1.17E-05	2.57E+13
NGC5548	7.718	5.23E+07	1.04E+38	3.83E+36	7.73E-05	3.88E+12
PG1426+015	9.007	1.02E+09	2.02E+39	6.09E+38	3.97E-06	7.55E+13
Mrk817	7.586	3.86E+07	7.67E+37	1.19E+37	1.05E-04	2.86E+12
Mrk290	7.277	1.89E+07	3.77E+37	5.26E+36	2.13E-04	1.41E+12
PG1613+658	8.339	2.18E+08	4.34E+38	2.01E+38	1.85E-05	1.62E+13

PG1617+175	8.667	4.64E+08	9.24E+38	2.21E+38	8.70E-06	3.45E+13
PG1700+518	8.786	6.10E+08	1.21E+39	2.67E+38	6.62E-06	4.53E+13
3C390.3	8.638	4.34E+08	8.64E+38	8.52E+37	9.30E-06	3.23E+13
1RXS J1858+4850	6.705	5.07E+06	1.01E+37	1.70E+36	7.96E-04	3.77E+11
Zw229-015	6.913	8.19E+06	1.63E+37	1.21E+37	4.93E-04	6.08E+11
NGC6814	7.038	1.09E+07	2.17E+37	2.83E+36	3.70E-04	8.11E+11
Mrk509	8.049	1.12E+08	2.23E+38	1.82E+37	3.61E-05	8.31E+12
PG2130+099	7.433	2.71E+07	5.39E+37	7.28E+36	1.49E-04	2.01E+12
NGC7469	1.80E+37	4.47E-04	1.80E+37	2.02E+36	4.47E-04	6.72E+11

Above is an exhaustive list of data obtained from the AGN Database, sponsored by the Georgia State university.

Calculation for obtaining the mass of a black hole which has detectable frequency, emitted through Hawking radiation. The average wavelength of the electromagnetic spectrum is 550 nm and will be used for the following calculation:

$$f = \frac{c^3}{16\pi GM}$$

Expressing the equation with wavelength,

$$\frac{c}{\lambda} = \frac{c^3}{16\pi GM}$$

Hence,

$$\frac{1}{\lambda} = \frac{c^2}{16\pi GM}$$

Giving,

$$\frac{16\pi GM}{c^2} = \lambda$$

Substituting the respective values for the constants
where G is 6.67×10^{-11}
where c is $3 \times 10^{8}\, ms^{-1}$
and λ is 500×10^{-9}

$$\frac{16\pi(6.67 \times 10^{-11})M}{(3 \times 10^{8})^2} = 500 \times 10^{-9}$$

$$M = 1.34 \times 10^{19}\, kgs$$

In terms of solar mass can be written as, $M_{sun} = 1.99 \times 10^{30}\ kgs$

$$M = \frac{1.34 \times 10^{19}}{1.99 \times 10^{30}}\, M_{sol}$$

$$M = 6.74 \times 10^{-12} M_{sol}$$

Since the mass of the black hole is significantly less then one solar mass, it can be classified as primordial. In conclusion, in order for the radiation emitted by black holes to be detectable (approximately equal to the average wavelength of EM spectrum - 550nm) the black holes must be of $M \leq 6.74 \times 10^{-12} M_{sol}$.

